Multiple-Site, Multiple-Mediator Instrumental Variables (MSMM-IV) Methods: Conceptual Model and Assumptions

sean f. reardon

July, 2016

Outline

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2. Intuition of the MSMM-IV model
3. Additional Examples
4. Formal Assumptions of the MSMM-IV model
5. Visualizing the MSMM-IV model
6. Examples of the MSMM-IV assumptions (and a discussion of the need for theory)
7. Study design considerations

A Motivating Example:

Suppose we offer families a voucher to pay for up to $5,000 for pre-school. We want to know if this improves children's school readiness.

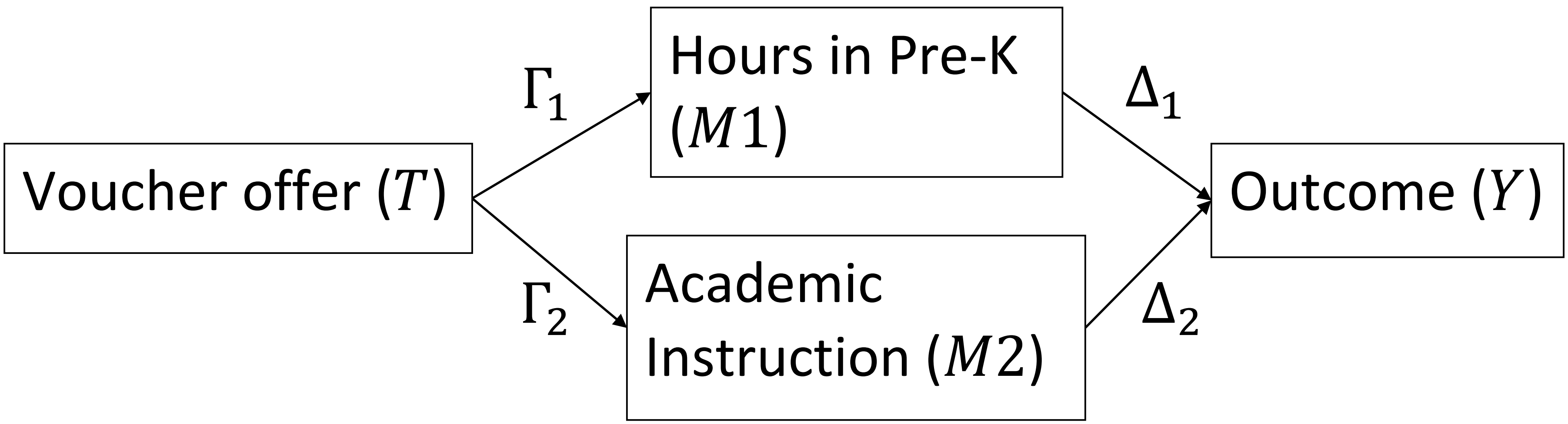
Under random assignment, it is straightforward to estimate the average effect of the voucher offer on school readiness.

In a multi-site trial, it may also be possible to estimate the variance in the effect of the voucher across sites.

But if we wish to know how the voucher affects readiness, **we need a theory, and a design that allows us to test that theory.**

A simple (simplistic) theory:

The voucher offer may induce a) more time in pre-school and/or b) the use of pre-K programs that focus on teaching academic skills.



How do we test this theory, and estimate the effects of pre-school time and type?

Suppose we conduct this experiment in a number of sites. In each site we can estimate both

a) The average effect of the voucher offer on school readiness ($\beta_s = E[B|site = s]$).
b) The average effects of the voucher offer on both time and type of pre-school:

$$\gamma_{1s} = E[\Gamma_1|site = s];\ \gamma_{2s} = E[\Gamma_2|site = s]$$

Next we estimate the partial association between β_s and γ_{1s} and γ_{2s}, via the regression model:

$$\beta_s = \delta_1(\gamma_{1s}) + \delta_2(\gamma_{2s}) + \omega_s$$

Here δ_1 describes the average difference in β between two sites that differ by one unit on γ_1, holding constant γ_2. Under some assumptions, $\delta_1$is the average effect of $M1$ on Y.

Example 2:

Duncan, Morris, Rodrigues (2011) use sixteen implementations of random-assignment welfare-to-work experiments (each with slightly different program designs) to estimate the impact of three hypothesized mediators of the programs: income, hours worked, and welfare receipt.

Within any site, it is impossible to use IV to estimate the effects of 3 mediators based on a single instrument (random assignment to program). But the replication of the study across 16 sites enables them to generate 16 instruments (site-by-assignment interactions) to estimate the effect of 3 hypothesized mediators.

Example 3:

In 5 cities, the MTO experiment randomly assigned low-income families to receive 1) a housing voucher to pay for housing in a low-poverty neighborhood; 2) a regular section 8 housing voucher; or 3) no voucher.

Kling, Liebman, and Katz (2007) use the study to estimate the effects of neighborhood poverty on child/family outcomes, controlling for the effect of moving (using the voucher). They rely on 10 instruments (5 sites interacted with 2 treatment variables) to identify the effect of the two mediators (neighborhood poverty and mobility).

The Multiple-site, Multiple-Mediator Instrumental Variables Model (MSMM-IV): Selected Papers

Duncan GJ, Morris PA, Rodrigues C. 2011. Does Money Really Matter? Estimating Impacts of Family Income on Young Children's Achievement with Data from Random-Assignment Experiments. *Developmental Psychology* 47: 1263-79

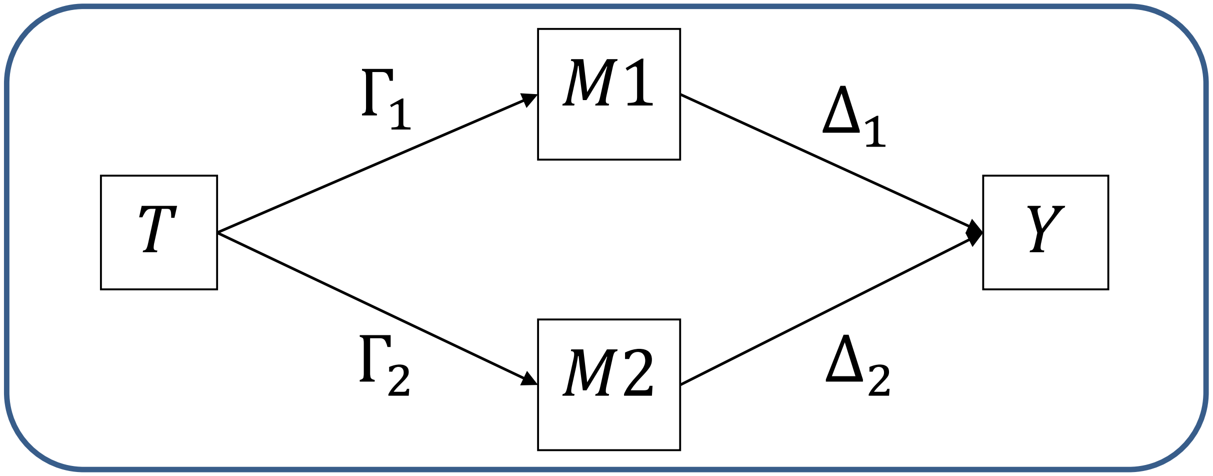
Kling JR, Liebman JB, Katz LF. 2007. Experimental Analysis of Neighborhood Effects. *Econometrica* 75: 83-119

Nomi T, Raudenbush, SW. 2016. Making a Success of "Algebra for All": The Impact of Extended Instructional Time and Classroom Peer Skill in Chicago *Educational Evaluation and Policy Analysis* 38(June): 431-451.

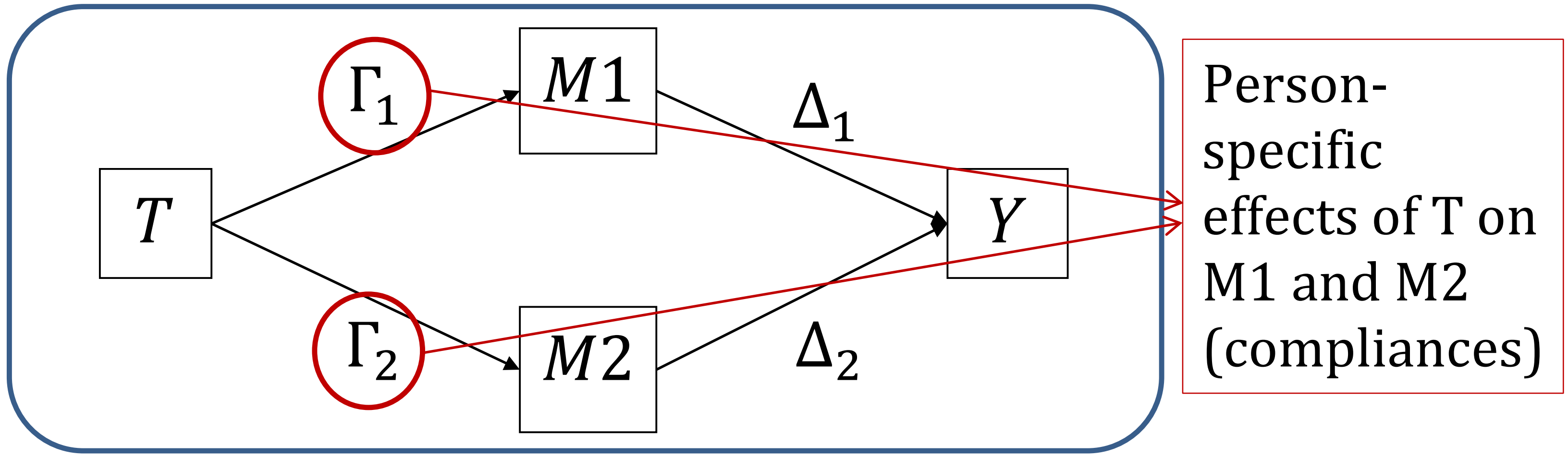
Reardon SF, Raudenbush SW. 2013. Under What Assumptions do Site-by-Treatment Instruments Identify Average Causal Effects? *Sociological Methods and Research.* 42(2): 143-163.

Reardon SF, Unlu F, Zhu P, Bloom H. 2013. Bias and Bias Correction in Multi-Site Instrumental Variables Analysis of Heterogeneous Mediator Effects. *Journal of Educational and Behavioral Statistics* 39(1): 53-86.

Site 1:

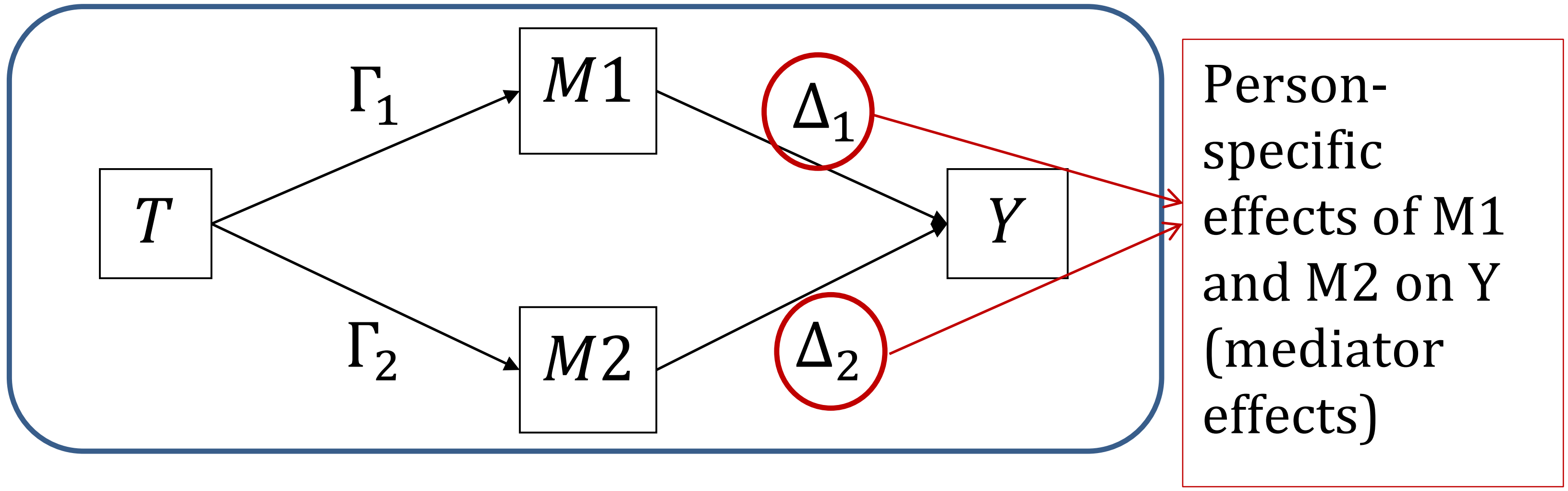


Site 1:

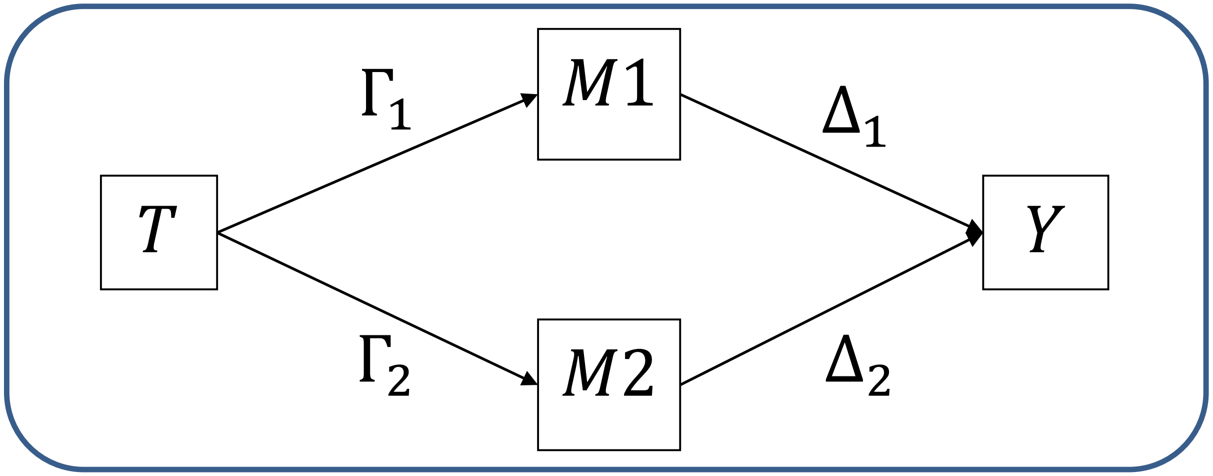


T
Γ1
Γ2
M1
M2
Δ1
Δ2
Y
Person-specific effects of T on M1 and M2 (compliances)

Site 1:


T
M1
M2
Y
Γ1
Γ2
Δ1
Δ2
Person-specific effects of M1 and M2 on Y (mediator effects)

Site 1:



The person specific effect of T on Y is

$$\mathrm{B} = \Delta_1\Gamma_1 + \Delta_2\Gamma_2$$

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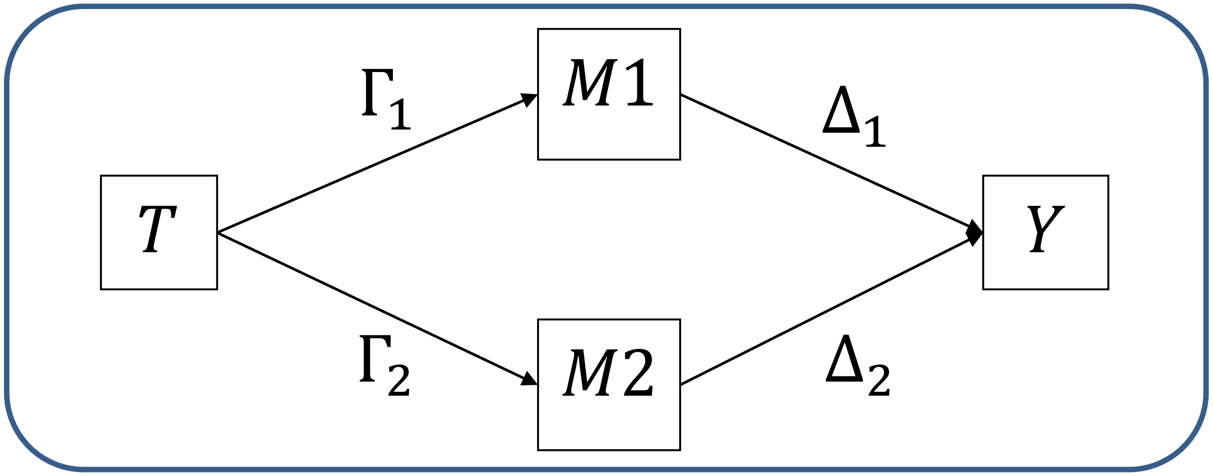


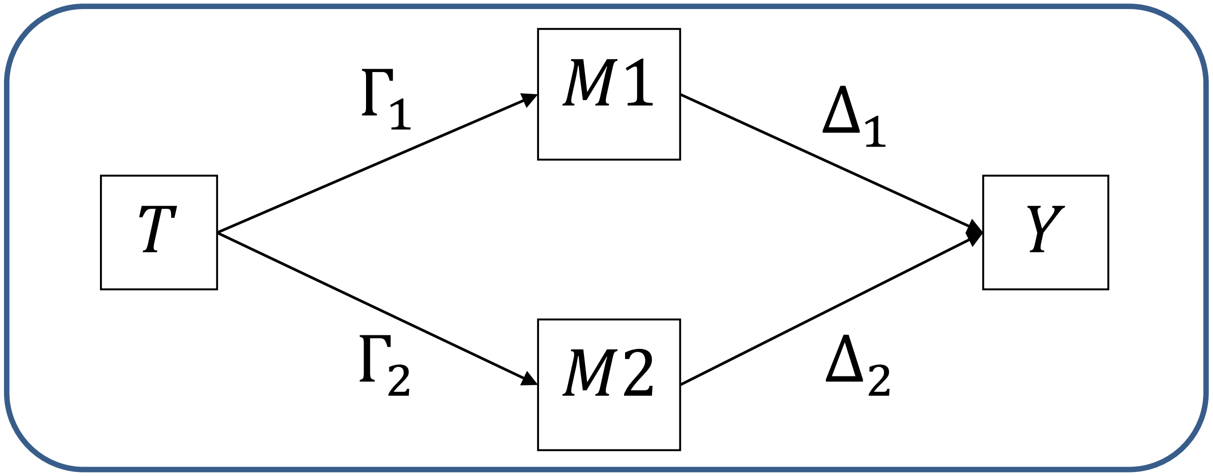
The person specific effect of T on Y is

$$B = \Delta_1\Gamma_1 + \Delta_2\Gamma_2$$

This contains several implicit assumptions:

1. SUTVA.
2. $\boldsymbol{M}$ is linear in $\boldsymbol{T}$.
3. Y is linear in $\boldsymbol{M}$.
4. The **mediators act in parallel**.
5. The **exclusion restriction** (no direct effect of T on Y.

Site 1:



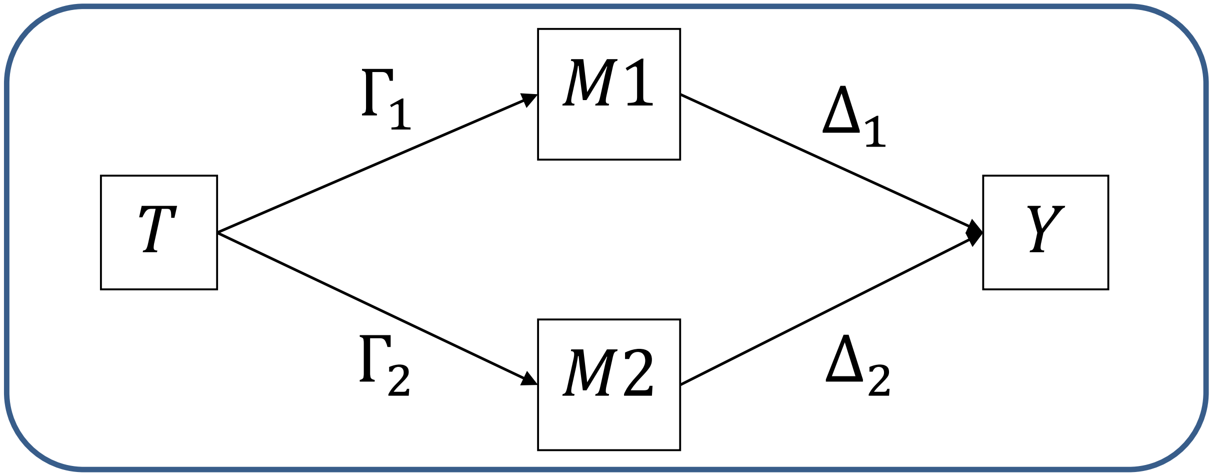
The person specific effect of T on Y is

$$\mathrm{B} = \Delta_1\Gamma_1 + \Delta_2\Gamma_2$$

Within this site, the average effect of T on Y is

$$\begin{aligned}\beta &= E[\mathrm{B}] \\ &= E[\Delta_1\Gamma_1 + \Delta_2\Gamma_2] \\ &= E[\Delta_1]E[\Gamma_1] + E[\Delta_2]E[\Gamma_2] + Cov(\Delta_1, \Gamma_1) + Cov(\Delta_1, \Gamma_2) \\ &= \delta_1\gamma_1 + \delta_2\gamma_2 + Cov(\Delta_1, \Gamma_1) + Cov(\Delta_1, \Gamma_2)\end{aligned}$$

Site 1:



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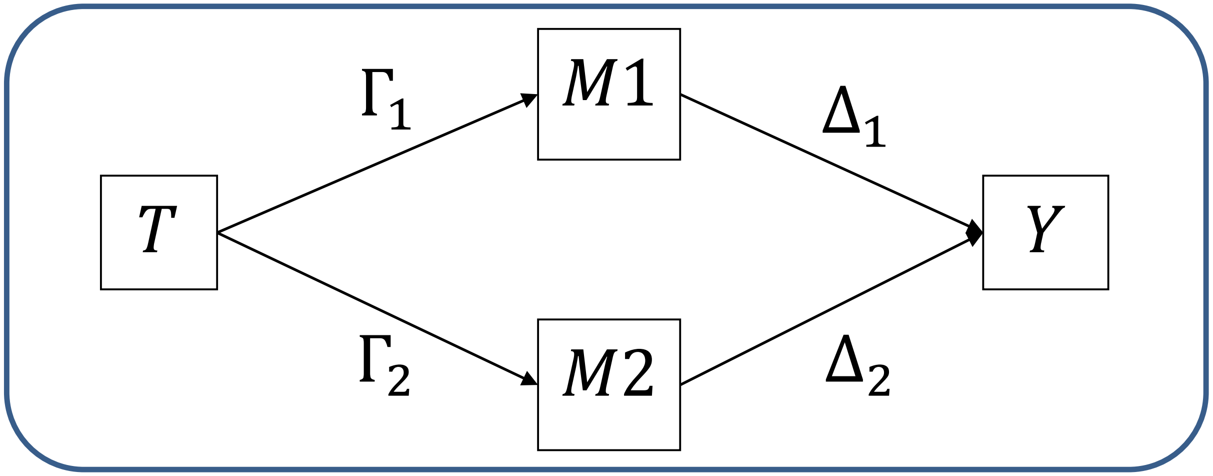
Within this site, the average effect of T on Y is

$$\begin{aligned}\beta &= E[\mathrm{B}] \\ &= E[\Delta_1\Gamma_1 + \Delta_2\Gamma_2] \\ &= E[\Delta_1]E[\Gamma_1] + E[\Delta_2]E[\Gamma_2] + Cov(\Delta_1,\Gamma_1) + \\ &= \delta_1\gamma_1 + \delta_2\gamma_2 + Cov(\Delta_1,\Gamma_1) + Cov(\Delta_1,\Gamma_2) \\ &= \boldsymbol{\delta_1}\gamma_1 + \boldsymbol{\delta_2}\gamma_2\end{aligned}$$

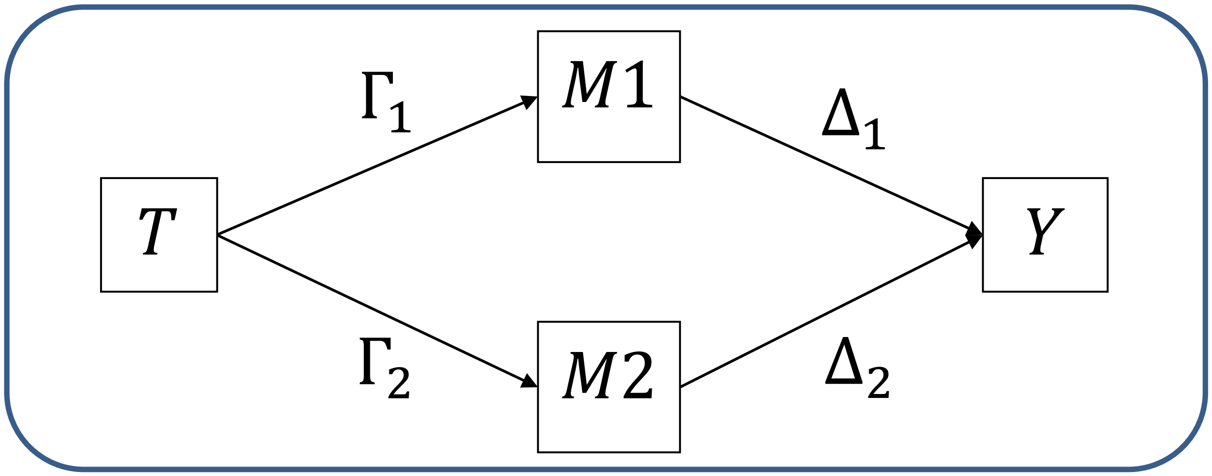
Assumption:

6. **No within-site compliance-effect covariance.**

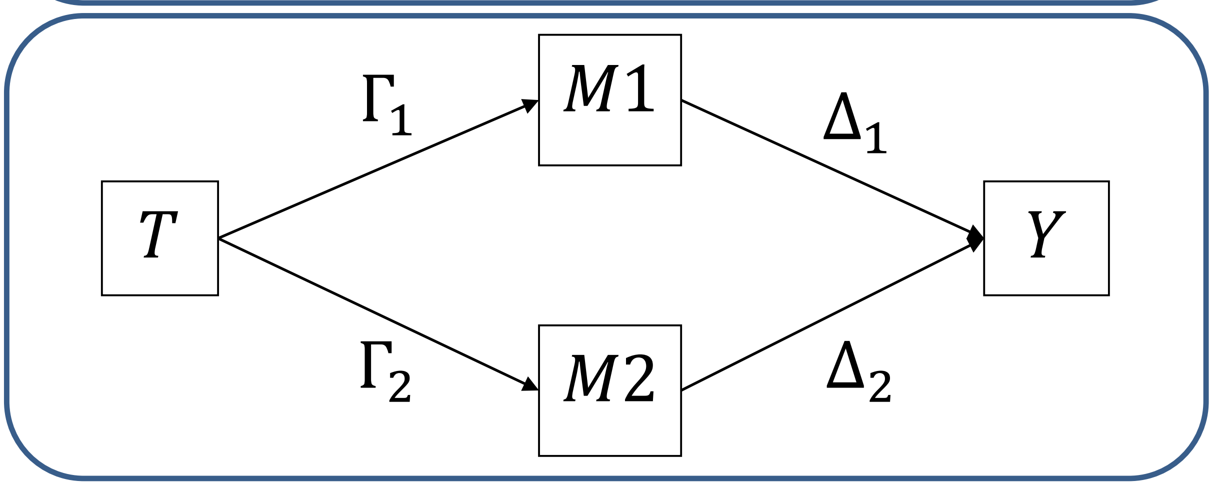
Site 1:



Site 2:



Site J:



Now, in each site s, under the assumptions above, we have

$$
\begin{aligned}
\beta_s &= \delta_{1s}\gamma_1 + \delta_{2s}\gamma_{2s} \\
&= \delta_1\gamma_{1s} + \delta_2\gamma_{2s} + (\delta_{1s} - \delta_1)\gamma_{1s} + (\delta_{2s} - \delta_2)\gamma_{2s} \\
&= \delta_1\gamma_{1s} + \delta_2\gamma_{2s} + \omega_s
\end{aligned}
$$

where δ_p is the average, across sites, of the δ_{ps}'s, and

$$\omega_s = (\delta_{1s} - \delta_1)\gamma_{1s} + (\delta_{2s} - \delta_2)\gamma_{2s}.$$

Now, in each site s, under the assumptions above, we have

$$\beta_s = \delta_{1s}\gamma_1 + \delta_{2s}\gamma_{2s}$$
$$= \delta_1\gamma_{1s} + \delta_2\gamma_{2s} + (\delta_{1s} - \delta_1)\gamma_{1s} + (\delta_{2s} - \delta_2)\gamma_{2s}$$
$$\boxed{\boldsymbol{\beta_s = \delta_1\gamma_{1s} + \delta_2\gamma_{2s} + \omega_s}}$$

where δ_p is the average, across sites, of the δ_{ps}'s, and $\omega_s = (\delta_{1s} - \delta_1)\gamma_{1s} + (\delta_{2s} - \delta_2)\gamma_{2s}$.

The site-average ITT effects (the β's) can be expressed as a linear combination of the site-average compliances (the γ_p's) plus some site-specific residual error term. The coefficients δ_1 and δ_2 on the γ_{ps} terms are the parameters of interest—the average effects of the mediators on the outcome Y.

This suggests we can estimate the $\boldsymbol{\delta}$'s via linear regression.

$$\boldsymbol{\beta_s = \delta_1 \gamma_{1s} + \delta_2 \gamma_{2s} + \omega_s}$$

We require three additional assumptions to identify the δ's from this regression model:

Additional assumptions:

7. Ignorable assignment of T within each site (this allows us to estimate the β_s's, γ_{1s}'s, and γ_{2s}'s from the observed data.
8. The J x P matrix of the $\gamma_{ps}'s$ has rank P (linear independence of the γ_p's).
9. Between-site compliance-effect independence **(the site-average compliance of each mediator is independent of the site-average effect of each mediator).** This ensures that the error $\boldsymbol{\omega_s}$ is independent: $\boldsymbol{E[\omega_s|\gamma_{1s}, \gamma_{2s}] = E[\omega_s] = 0}$.

Summary of MSMM-IV Assumptions

1) Stable unit treatment value assumptions
2) Person-specific linearity of the mediators with respect to the treatment
3) Person-specific linearity of the outcome with respect to the mediators
4) Parallel mediators
5) Exclusion restriction
6) Zero within-site compliance-effect covariance for each mediator
7) Within-site ignorable treatment assignment
8) Compliance matrix has sufficient rank
9) Between-site cross-mediator compliance-effect independence

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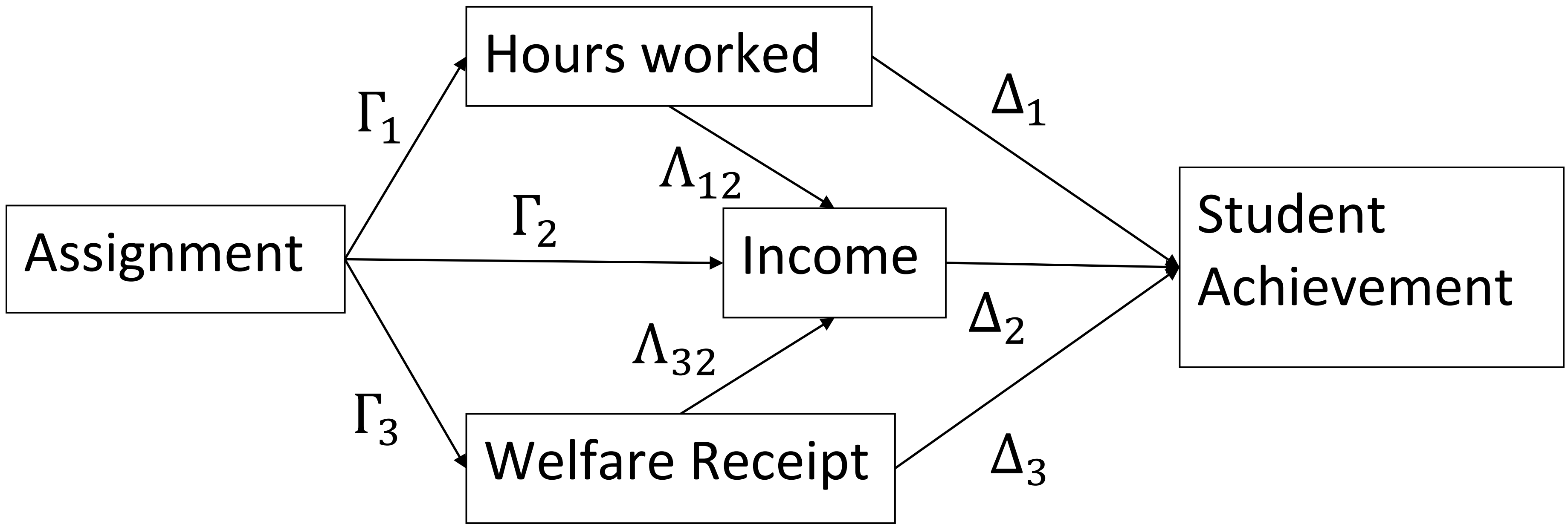
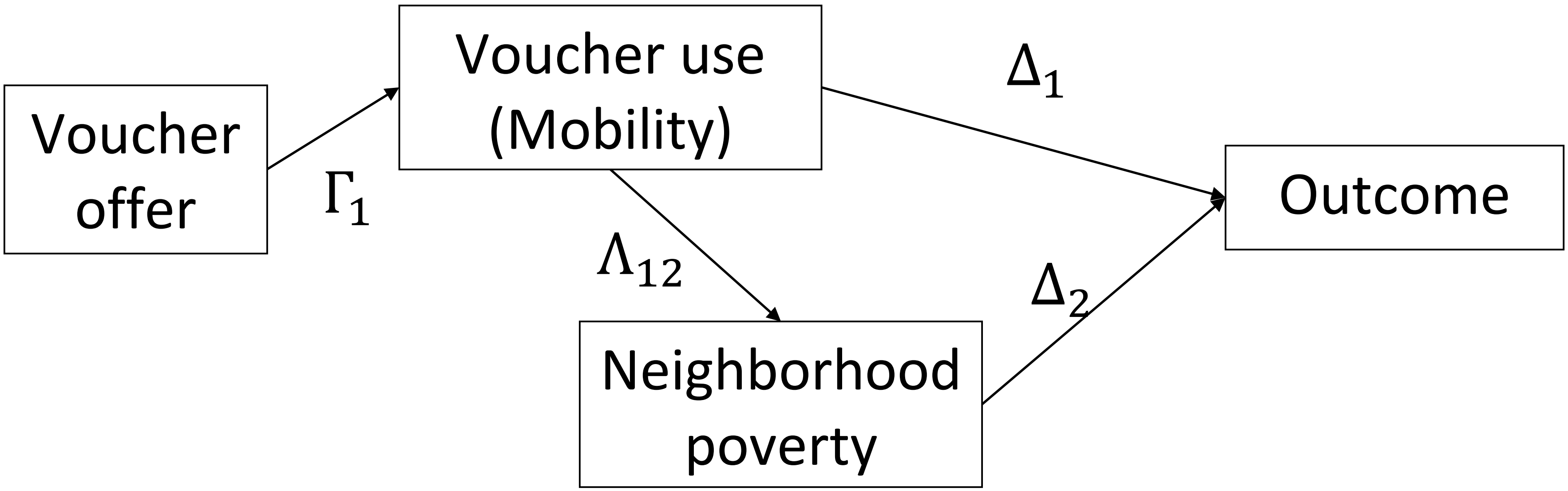
Examples of the assumptions

Duncan, Morris, Rodrigues (2011):



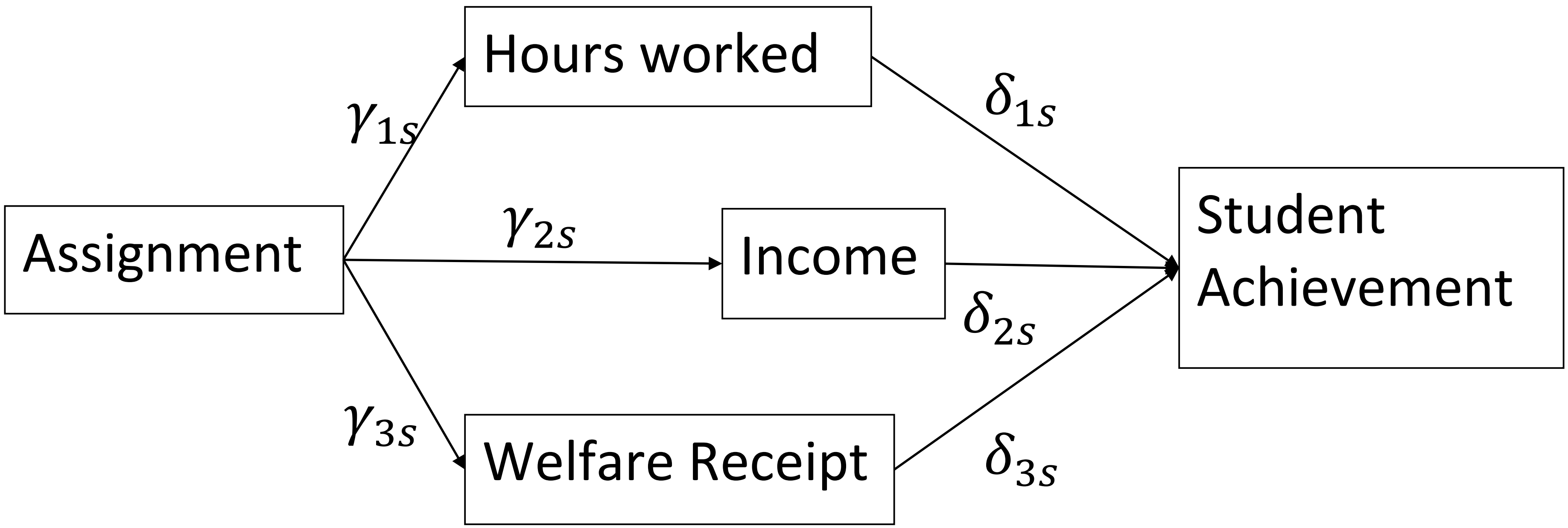
The parallel mediator assumption requires $\Lambda_{12} = \Lambda_{32} = 0$. It is possible to relax this assumption, but it requires another set of assumptions regarding the covariances among the Γ's, Λ's, and Δ's, and among the γ's, λ's, and δ's.

Kling, Liebman, & Katz (2007):



In MTO, the mediators are clearly not parallel, but it is still possible to estimate δ_2, given some additional assumptions about the relevant covariances.

Compliance effect independence assumption in Duncan et al:



Compliance-effect independence assumption requires that each γ_p is independent of all δ_q's. **We need a theoretical model describing the sources of variation across sites in the $\boldsymbol{\gamma}$'s and the $\boldsymbol{\delta}$'s (and their covariance).**

Why might γ's vary across sites?

- Different treatment design or implementation (treatment design/assignment differences across sites induce differential compliances – e.g., welfare to work programs had different designs and targeted different mediators).
- Different control conditions (different counterfactual mediator values across sites will affect γ's).
- Different populations (maybe some sites have very 'compliant' people; others have less 'compliant' people).
- Contextual differences affect compliance (e.g., local entry-level wages affect compliance with employment).
- Etc.

Similarly, why might δ's vary across sites?

- Different implementation of mediators (e.g., Head Start centers have different practices in different sites).
- Different populations (mediators more/less effective with different populations).
- Contextual factors moderate the impacts of the mediators (e.g., local cost of living may moderate the effect of income)
- Etc.

Why might γ's and δ's be non-independent?

- Selection on expected effects (i.e., in sites where people believe the effect of a mediator will be large, treatment assignment induces a larger change in the mediator).
- Common contextual conditions affect both compliance and effect (e.g., in sites with high cost of living, wages are high but costs are high, so income compliance is large and income effects are low – negative correlation between compliance and effect).
- Populations differ across sites (high-need populations may have higher/lower compliance, and effects may differ by population needs)
- Etc.

Concerns:

- These models rely on a large number of assumptions
    - Parallel mediators
    - Exclusion restriction
    - Between-site compliance-effect independence
- Need good theory/prior evidence to justify
- May need large number of sites

However…

- Bias due to compliance-effect non-independence may not be large (Reardon, Unlu, Zhu, & Bloom, forthcoming), and can be corrected.
- Exclusion restriction can be (partially) tested.

Visualizing the MSMM-IV model with a single mediator:

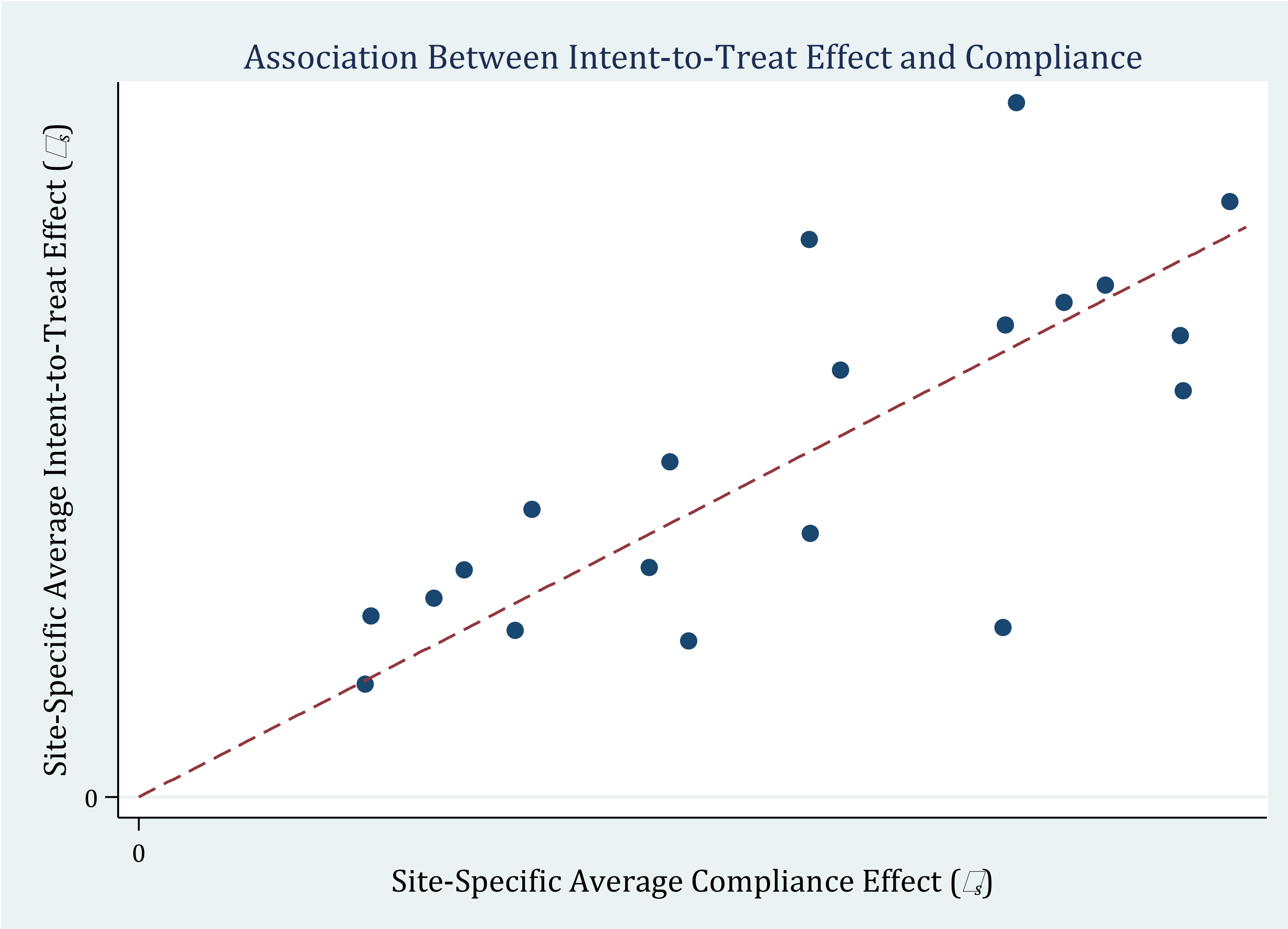
Recall that the model is:

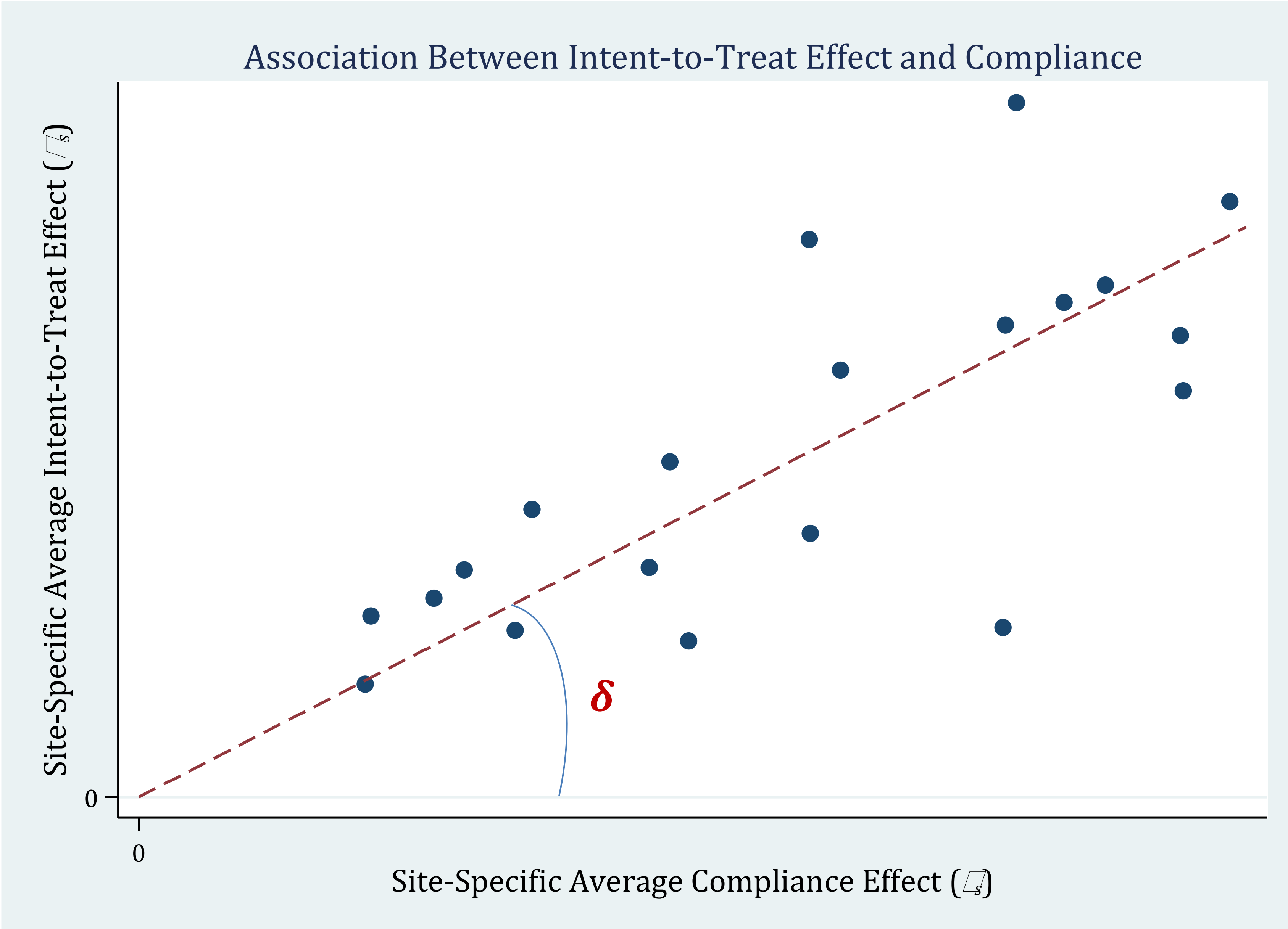
$$\beta_s = \delta_1 \gamma_{1s} + \omega_s$$

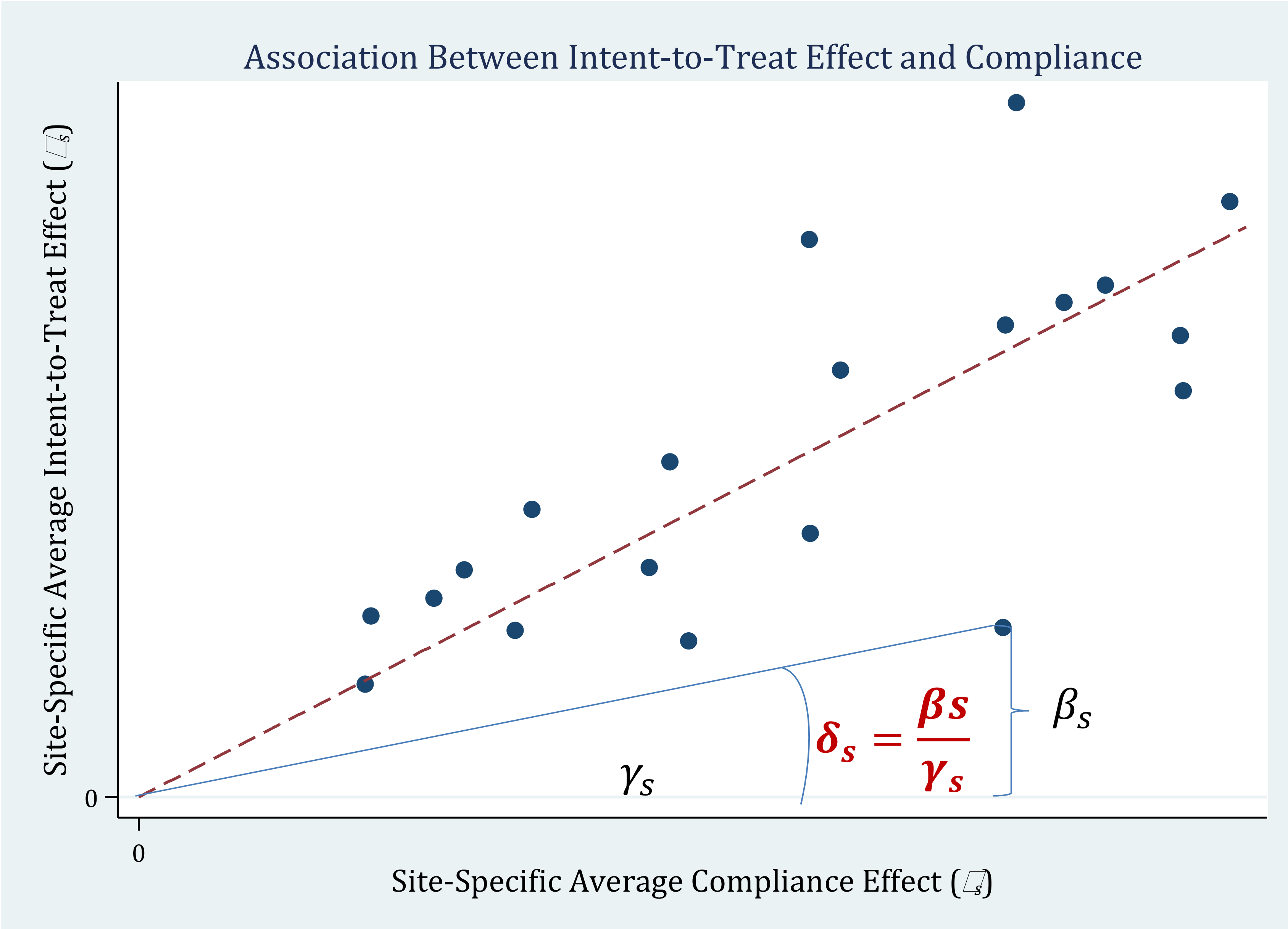
where the error

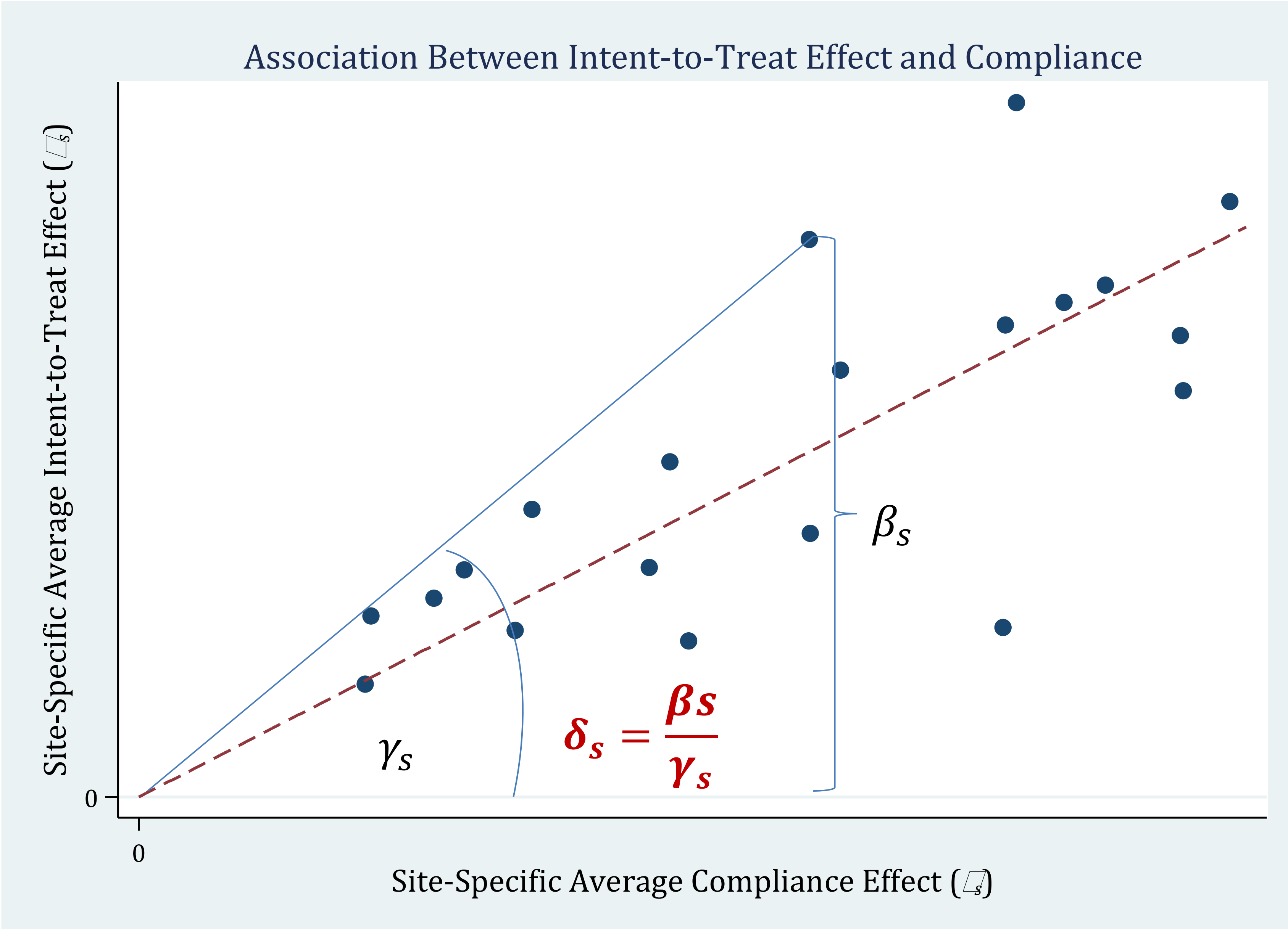
$$\omega_s = (\delta_{1s} - \delta_1)\gamma_{is}$$

is heteroskedastic (because it depends on γ_{1s}).


Association Between Intent-to-Treat Effect and Compliance
Site-Specific Average Intent-to-Treat Effect
0
0
Site-Specific Average Compliance Effect


Association Between Intent-to-Treat Effect and Compliance
Site-Specific Average Intent-to-Treat Effect
0
δ
0
Site-Specific Average Compliance Effect


Association Between Intent-to-Treat Effect and Compliance
Site-Specific Average Intent-to-Treat Effect
0
0
Site-Specific Average Compliance Effect
γ_s
$\delta_s = \frac{\beta s}{\gamma_s}$
β_s



Association Between Intent-to-Treat Effect and Compliance
Site-Specific Average Intent-to-Treat Effect (β_s)
Site-Specific Average Compliance Effect (γ_s)
0
0
β_s
γ_s
$\boldsymbol{\delta_s = \frac{\beta s}{\gamma_s}}$

Finite Sample Bias in the MSMM-IV Model

Under the MSMM-IV assumptions, δ is the coefficient in the model:

$$\beta_s = \delta\gamma_s + \omega_s.$$

However, the feasible regression (because we must estimate the β_s's and γ_s's) is:

$$\hat{\beta}_s = \delta\hat{\gamma}_s + \omega_s.$$

The error in $\hat{\gamma}_s$ will result in bias in $\hat{\delta}$. This is a version of what is called "finite sample bias" – bias that results from the fact that $\hat{\gamma}_s$ contains error (and this error may be correlated with the error in $\hat{\beta}_s$).

Weighting in the MSMM-IV model

The model we fit is

$$\begin{aligned}\hat{\beta}_s &= \delta\hat{\gamma}_s + \omega_s \\ &= \delta\hat{\gamma}_s + [(\delta_s - \delta)\hat{\gamma}_s + b_s]\end{aligned}$$

If δ_s is constant, then this is

$$\hat{\beta}_s = \delta\hat{\gamma}_s + b_s; \quad b_s \sim \left(0, se\left(\hat{\beta}_s\right)^2\right)$$

So, if we assume δ_s is constant, we can fit the model via WLS, with weights $w_s = se\left(\hat{\beta}_s\right)^{-2}$.

MSMM-IV is identical to 2SLS with site-by-instrument interactions (if $\boldsymbol{\delta_s}$ is constant across sites)

We have P first-stage models:

$$M_{1is} = \theta_{1s} + \gamma_{1s} T_{is} + u_{1is}$$
$$\vdots$$
$$M_{Pis} = \theta_{Ps} + \gamma_{Ps} T_{is} + u_{Pis}$$

Second-stage model:

$$Y_{is} = \lambda_s + \sum_{p=1}^{P} \delta_p \widehat{M}_{pis} + v_{is}$$

Reduced-form model:

$$Y_{is} = \alpha_s + \beta_s T_{is} + \omega_{is}$$

The $\hat{\delta}_p$'s from this model are identical to those from the MSMM-IV WLS regression of the $\hat{\beta}_s$'s on the vector of $\hat{\gamma}_{ps}$'s.

However, the 2SLS formulation assumes the δ_p's are constant across sites ($\delta_{p1} = \delta_{p2} = \cdots = \delta_{pJ} = \delta_p$); the MSMM-IV model does not.

As a result, the standard errors from the 2SLS model will be incorrect (too small), because they do not take into account the variance of the δ_{ps}'s across sites.

Weighting the MSMM-IV model if $\boldsymbol{\delta_s}$ is not assumed constant

The model we fit is

$$\begin{aligned}\hat{\beta}_s &= \delta\hat{\gamma}_s + [(\delta_s - \delta)\hat{\gamma}_s + b_s] \\ &= \delta\hat{\gamma}_s + \omega_s\end{aligned}$$

where

$$\begin{aligned}\omega_s &\sim \left(0, (\gamma_s^2 + se(\hat{\gamma}_s)^2)\tau_\delta + se\left(\hat{\beta}_s\right)^2\right) \\ &\approx \left(0, \tau + se\left(\hat{\beta}_s\right)^2\right)\end{aligned}$$

So we can fit this using precision weights equal to

$$w_s = \left(\tau + se\left(\hat{\beta}_s\right)^2\right)^{-1}$$