Multisite Causal Mediation Analysis

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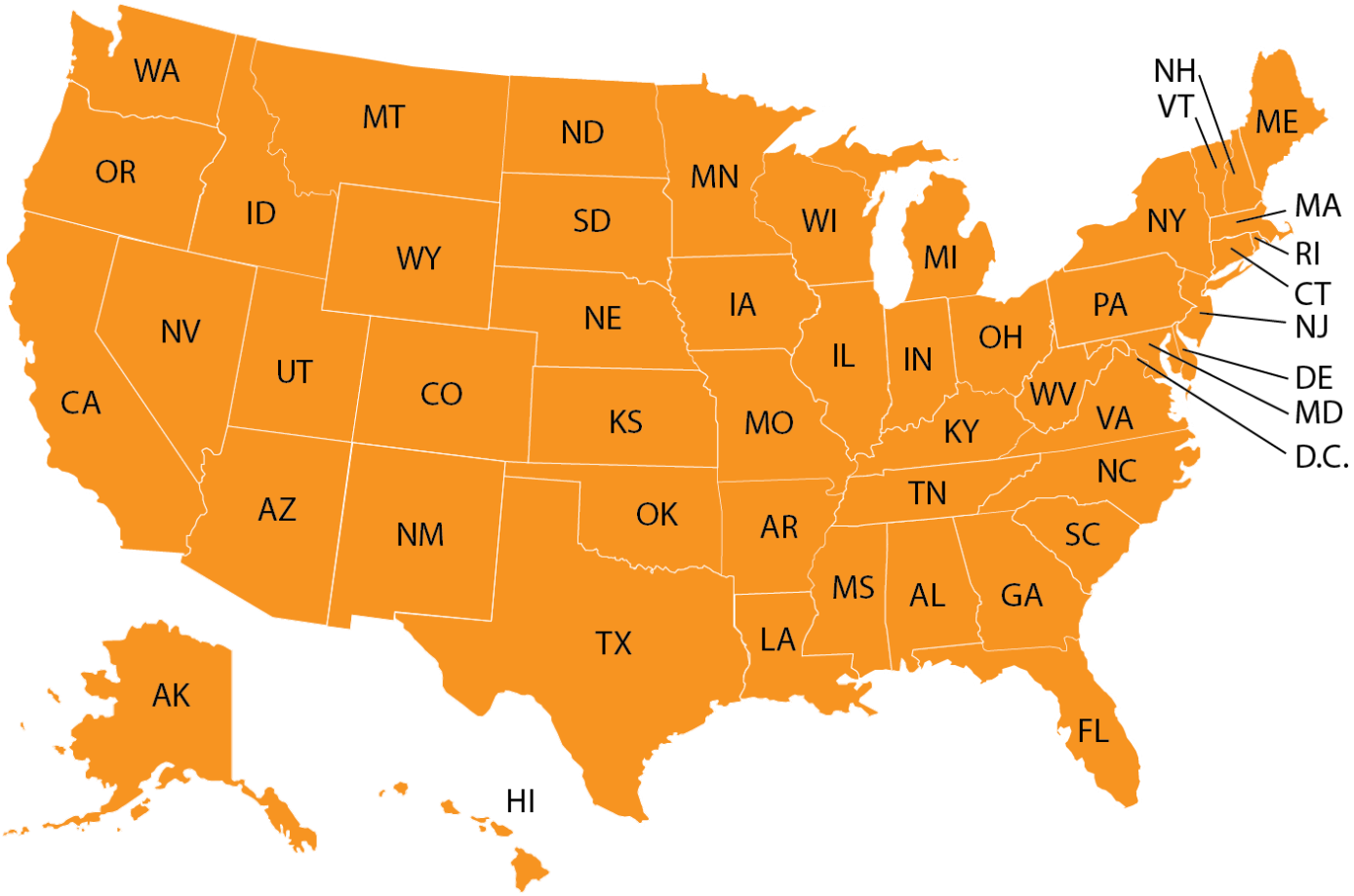
Outline

- National Job Corps Study
- Research Questions
- Methodological Challenges
- Identification
- Estimation
- Strengths and limitations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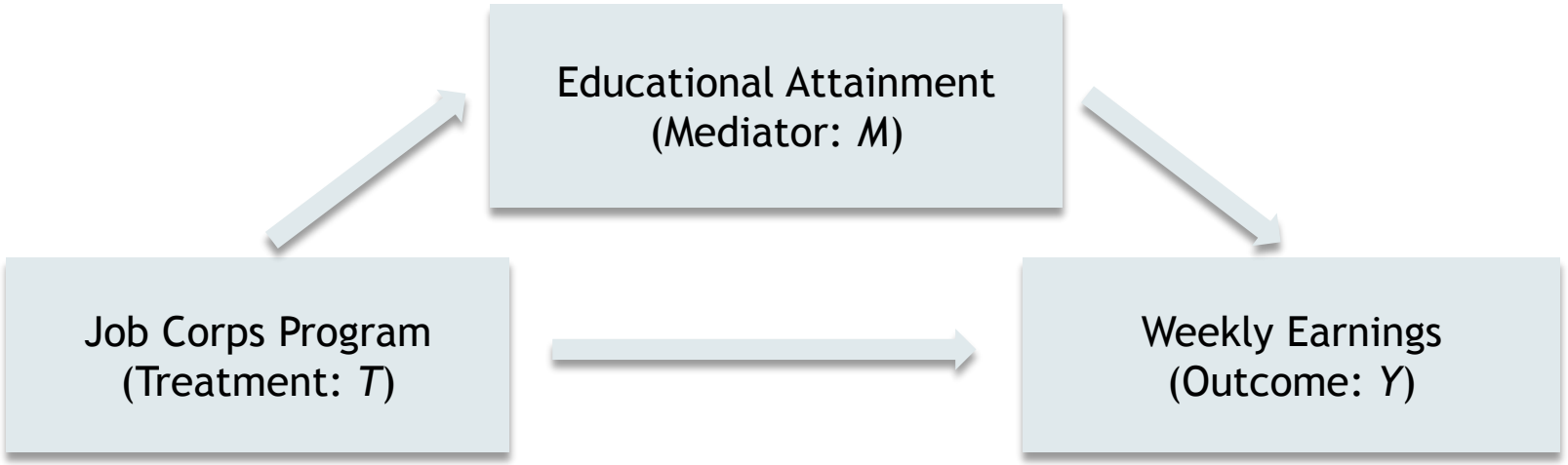
National Job Corps Study

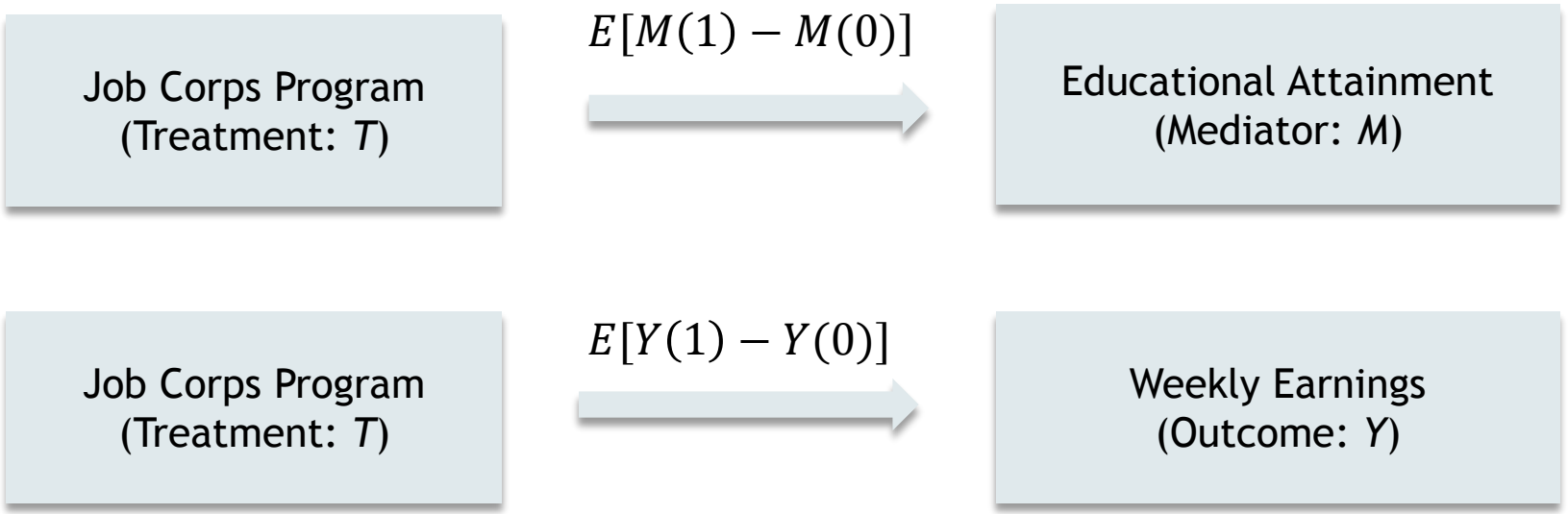
- **Job Corps**: the largest training to promote economic independence among disadvantaged youths ages 16-24 who are disconnected from school and work.
- **National Job Corps Study**: a multi-site randomized evaluation of Job Corps programs nationwide
- **Sample Size**: 9,409 applicants were randomly assigned to the experimental group and 5,977 to the control group. A total of 103 Job Corps centers served as the experimental sites. The number of applicants at each site ranged from 34 to 656.
- **Previous Findings**: on average, attaining an education or vocational training credential through the program mediated the Job Corps impacts on employment and earnings.

Hypothesized Mediation Mechanism

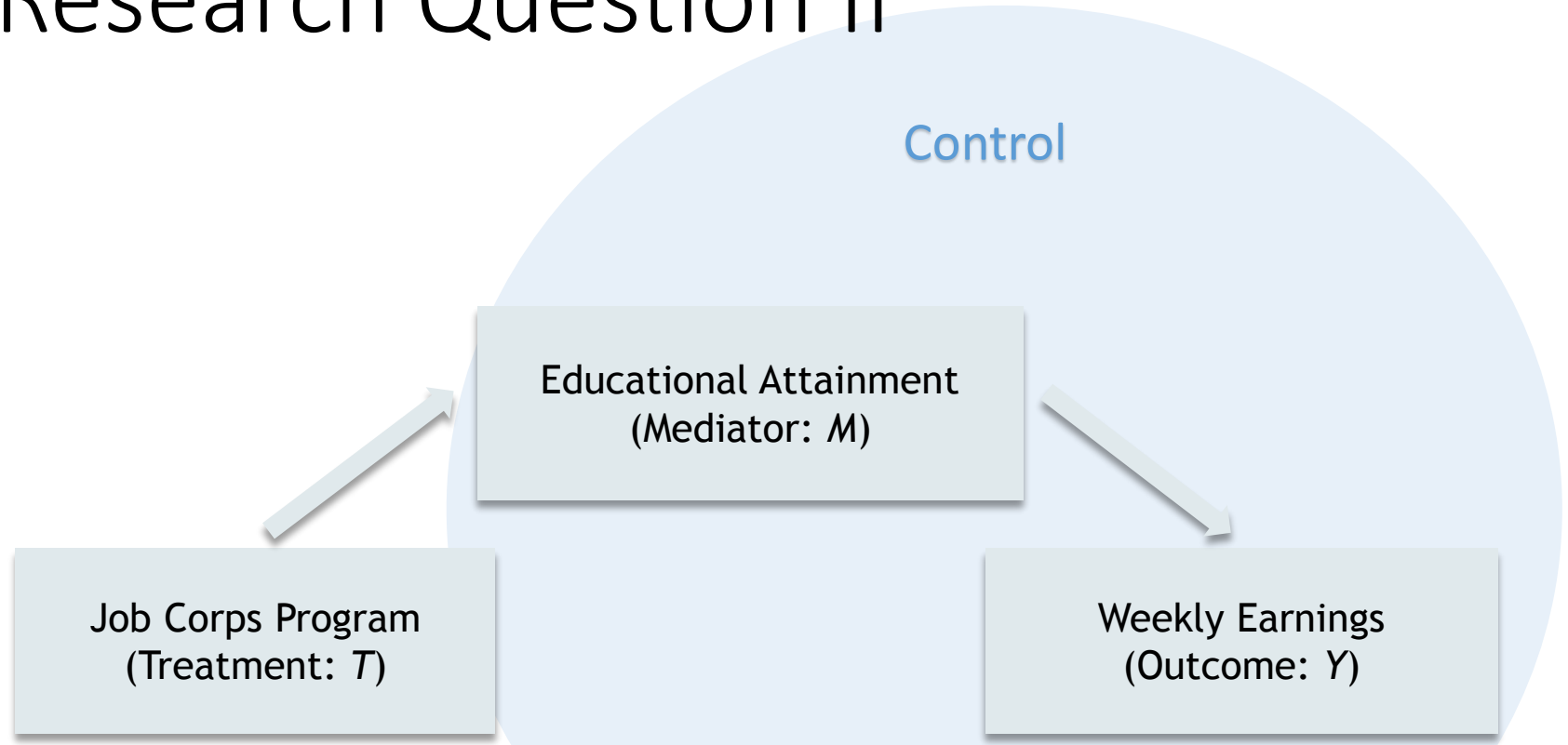


- **Mediator**: whether obtained an education/training credential during 30 months
- **Outcome**: weekly earnings in the 4th year after randomization
- Supplementary services: residential living, social skill training, individual counseling, drug and alcohol treatment, and health care

Research Question I

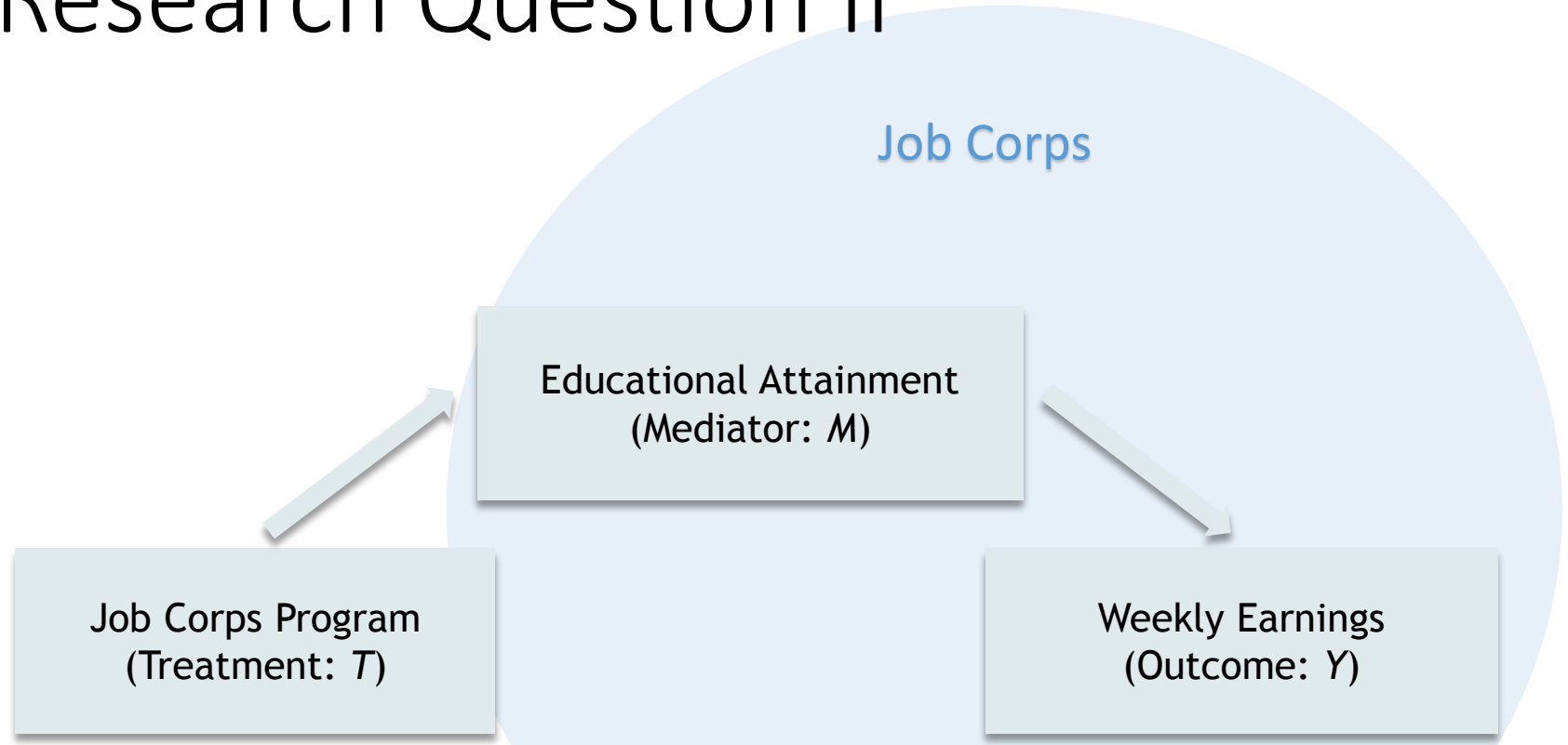


Research Question II



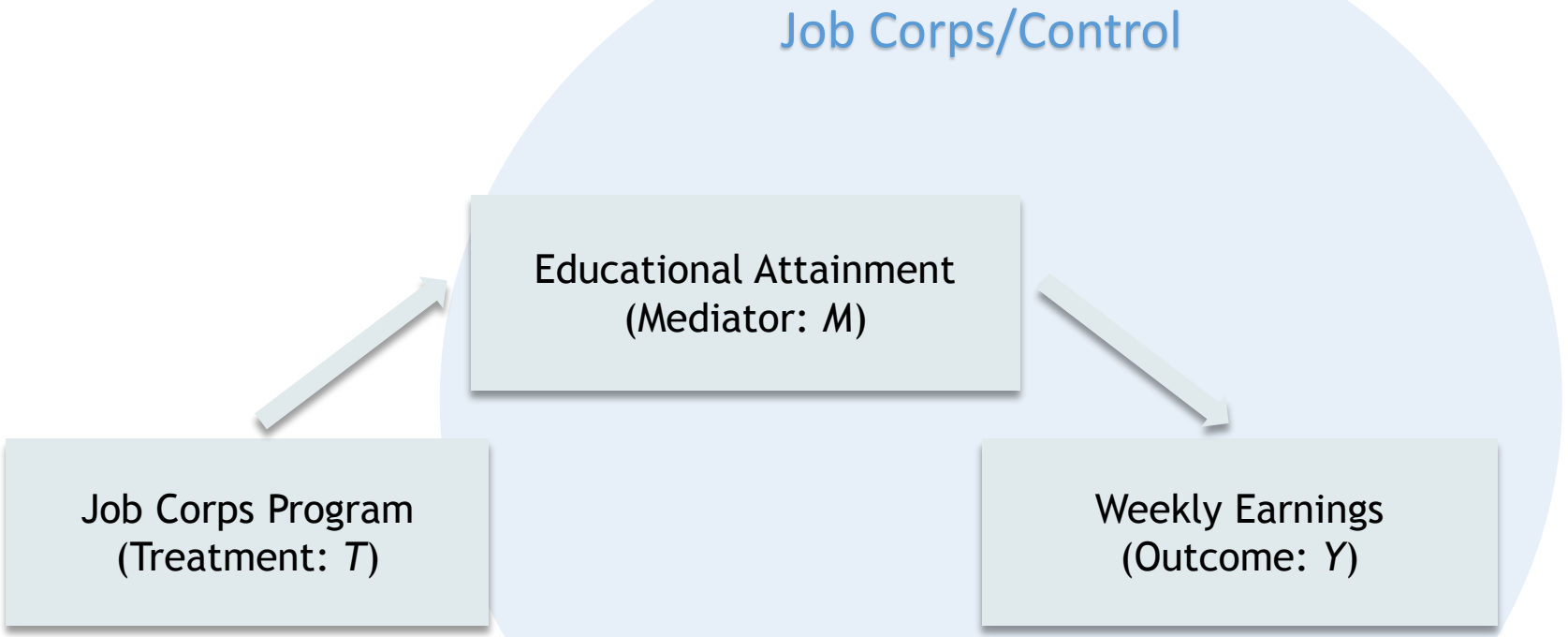
- How would the program-induced change in educational attainment exert an impact on earnings under the control condition (pure indirect effect) on average?
- How does it vary across sites and how is it correlated with site-specific program impact on educational attainment?

Research Question II



- How would the program-induced change in educational attainment exert an impact on earnings under JC condition (natural indirect effect) on average?
- How does it vary across sites and how is it correlated with site-specific program impact on educational attainment?

Research Question II



- The effect of program-induced change in education and training could be enhanced by the other supports provided by Job Corps that were generally unavailable under the control condition.
- If the above two are different, the extent to which this discrepancy (natural treatment-by-mediator interaction effect) was consistent across the sites.

Definition

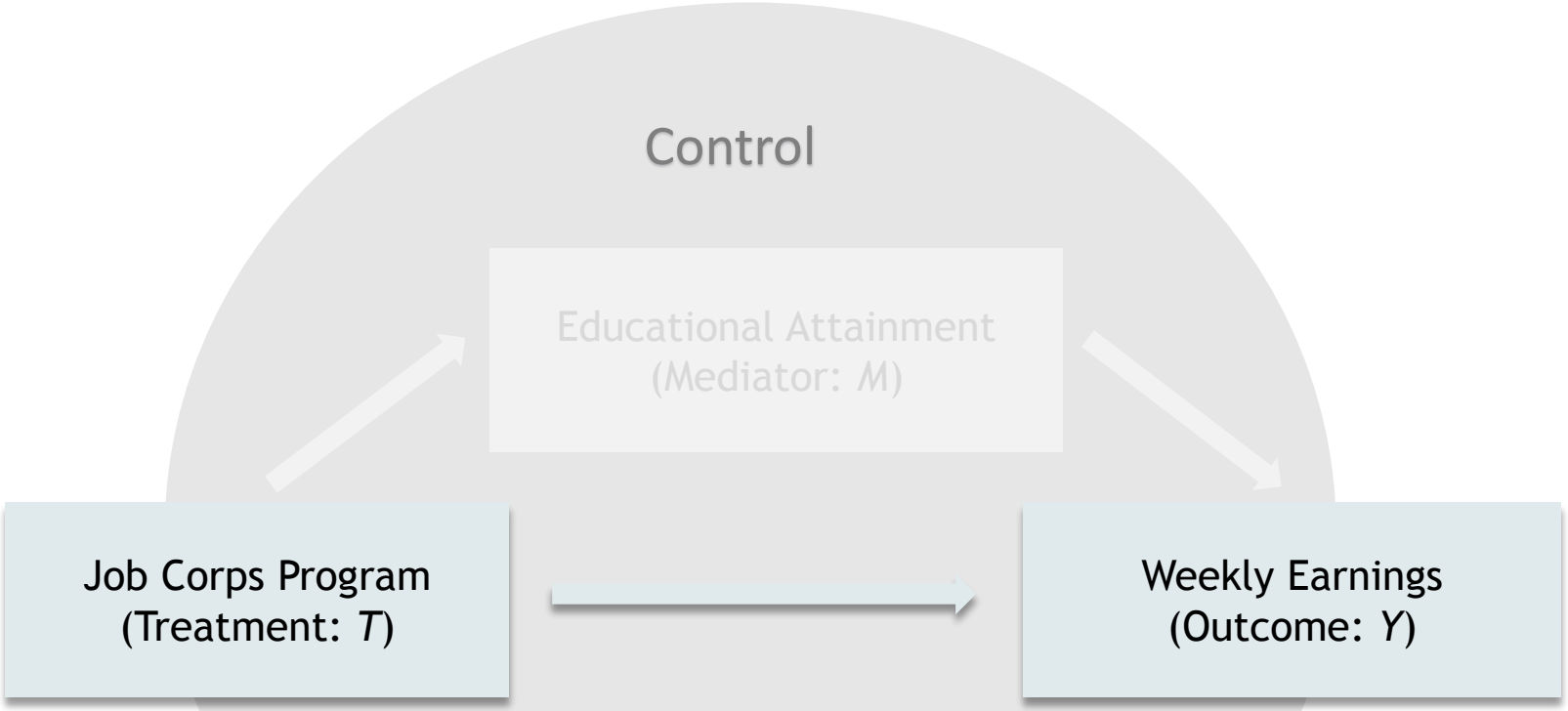
	Job Corps $T = 1$	Control $T = 0$
Educational attainment if assigned to Job Corps $M(1)$	$E[Y(1, M(1))]$	$E[Y(0, M(1))]$
Educational attainment if assigned to control $M(0)$	$E[Y(1, M(0))]$	$E[Y(0, M(0))]$

Natural Indirect Effect

Pure Indirect Effect

Natural Treatment-by-Mediator Effect

Research Question III



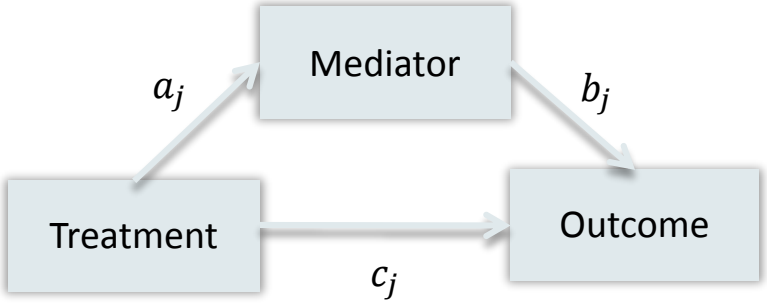
- How does the natural direct effect, which reflects the amount of variation across the sites in the relative contribution to the program impact of unspecified pathways vary across sites?
- How is it correlated with site-specific natural indirect effect?

Definition

	Job Corps $T = 1$	Control $T = 0$
Educational attainment if assigned to Job Corps $M(1)$	$E[Y(1, M(1))]$	$E[Y(0, M(1))]$
Educational attainment if assigned to control $M(0)$	$E[Y(1, M(0))]$	$E[Y(0, M(0))]$

Natural Direct Effect

Path Analysis Models For Multi-site Data



$$M_{ij} = (d_M + u_{Mj}) + (a + u_{aj})T_{ij} + e_{Mij}$$

$$Y_{ij} = (d_Y + u_{Yj}) + (c + u_{cj})T_{ij} + (b + u_{bj})M_{ij} + e_{Yij}$$

$$\begin{pmatrix} e_{Mij} \\ e_{Yij} \\ u_{Mj} \\ u_{a_j} \\ u_{Y_j} \\ u_{c_j} \\ u_{b_j} \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_{e_M} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma^2_{e_Y} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \tau^2_M & \tau_{M,a} & \tau_{M,Y} & \tau_{M,c} & \tau_{M,b} \\ 0 & 0 & \tau_{M,a} & \tau^2_a & \tau_{a,Y} & \tau_{a,c} & \tau_{a,b} \\ 0 & 0 & \tau_{M,Y} & \tau_{a,Y} & \tau^2_Y & \tau_{Y,c} & \tau_{Y,b} \\ 0 & 0 & \tau_{M,c} & \tau_{a,c} & \tau_{Y,c} & \tau^2_c & \tau_{c,b} \\ 0 & 0 & \tau_{M,b} & \tau_{a,b} & \tau_{Y,b} & \tau_{c,b} & \tau^2_b \end{pmatrix} \right]$$

- Population Average Indirect Effect: $ab + \tau_{a,b}$
- Between-Site Variance of Indirect Effect: $b^2\tau_a^2 + a^2\tau_b^2 + \tau_a^2\tau_b^2 + 2ab\tau_{a,b} + \tau_{a,b}^2$

Challenges to Path Analysis

- Rely on correct specifications of both the mediator model and the outcome model including
  - the functional forms
  - the distributional forms
- Because the average indirect effect is represented as a combination of multiple parameters, it adds considerable complications to estimation and statistical inference especially when research interests lie in
  - the between-site variance of the indirect effect
  - the covariance between the site-specific direct effect and indirect effect
- Their applications to discrete mediators and outcomes are particularly limited in both single-site and multisite studies

Counterfactual Framework

	Job Corps $T_{ij} = 1$	Control $T_{ij} = 0$
Educational attainment if assigned to Job Corps $M_{ij}(1)$	$E[Y_{ij}(1, M_{ij}(1))]$	$E[Y_{ij}(0, M_{ij}(1))]$
Educational attainment if assigned to control $M_{ij}(0)$	$E[Y_{ij}(1, M_{ij}(0))]$	$E[Y_{ij}(0, M_{ij}(0))]$

Under SUTVA:

No interference between units within a site

No interference between sites

Identification Assumptions

- Sequential randomization within levels of pretreatment covariates at each site:
  - Within levels of the observed pretreatment covariates, the treatment assignment was randomized at each site.
  - Within levels of pretreatment covariates, the mediator value assignment was also randomized under each treatment at each site.

Identification Results

	Job Corps $T_{ij} = 1$	Control $T_{ij} = 0$
Educational attainment if assigned to Job Corps $M_{ij}(1)$	$E[Y_{ij}(1, M_{ij}(1))\|S_{ij} = j]$ $\|\|$ $E[Y_{ij}\|T_{ij} = 1, S_{ij} = j]$	$E[Y_{ij}(0, M_{ij}(1))\|S_{ij} = j]$ $\|\|$ $E[W'_{ij}Y_{ij}\|T_{ij} = 0, S_{ij} = j]$
Educational attainment if assigned to control $M_{ij}(0)$	$E[Y_{ij}(1, M_{ij}(0))\|S_{ij} = j]$ $\|\|$ $E[W_{ij}Y_{ij}\|T_{ij} = 1, S_{ij} = j]$	$E[Y_{ij}(0, M_{ij}(0))\|S_{ij} = j]$ $\|\|$ $E[Y_{ij}\|T_{ij} = 0, S_{ij} = j]$

$$W_{ij} = \frac{\Pr(M_{ij} = m|T_{ij} = 0, \boldsymbol{X}_{ij} = \boldsymbol{x}, S_{ij} = j)}{\Pr(M_{ij} = m|T_{ij} = 1, \boldsymbol{X}_{ij} = \boldsymbol{x}, S_{ij} = j)}$$

$$W'_{ij} = \frac{\Pr(M_{ij} = m|T_{ij} = 1, \boldsymbol{X}_{ij} = \boldsymbol{x}, S_{ij} = j)}{\Pr(M_{ij} = m|T_{ij} = 0, \boldsymbol{X}_{ij} = \boldsymbol{x}, S_{ij} = j)}$$

Analytic Procedure

- Step 1 Propensity score estimation
- Step 2 Site-specific effect estimation
- Step 3 Population average effect estimation
- Step 4 Between-site variance and covariance estimation
- Step 5 Sensitivity analysis

Step 1 Propensity score estimation

Let $p_{tij}^{(t)} = Pr(M_{ij} = 1 | T_{ij} = t, \boldsymbol{X}_{ij} = \boldsymbol{x}, S_{ij} = j)$,

$$logit\left(\frac{p_{tij}^{(t)}}{1 - p_{tij}^{(t)}}\right) = \boldsymbol{X}'_{tij}\boldsymbol{\alpha}_t + \boldsymbol{S}'_{tij}\boldsymbol{r}_{tj}, \boldsymbol{r}_{tj} \sim N(\boldsymbol{0}, \boldsymbol{\Sigma}_t), t = 0,1$$

- $\boldsymbol{X}$: age, gender, race, criminal involvement, drug use, employment, and earnings at the baseline. $\boldsymbol{S}$: vector of covariates with random effects $\boldsymbol{r}_j$.
- Balance checking: assess whether the distributions of $\boldsymbol{X}$ are similar across combinations of the treatment and mediator levels at all the sites after controlling for the pretreatment covariates.

Step 2 Site-specific effect estimation

	Job Corps $T_{ij} = 1$	Control $T_{ij} = 0$
Educational attainment if assigned to Job Corps $M_{ij}(1)$	$E[Y_{ij}\|T_{ij} = 1, S_{ij} = j]$ $\uparrow$ $\frac{\sum_{i=1}^{n_j} T_{ij}Y_{ij}}{\sum_{i=1}^{n_j} T_{ij}}$	$E[W'_{ij}Y_{ij}\|T_{ij} = 0, S_{ij} = j]$ $\uparrow$ $\frac{\sum_{i=1}^{n_j} \widehat{W}'_{ij}(1 - T_{ij})Y_{ij}}{\sum_{i=1}^{n_j} \widehat{W}'_{ij}(1 - T_{ij})}$
Educational attainment if assigned to control $M_{ij}(0)$	$E[W_{ij}Y_{ij}\|T_{ij} = 1, S_{ij} = j]$ $\uparrow$ $\frac{\sum_{i=1}^{n_j} \widehat{W}_{ij}T_{ij}Y_{ij}}{\sum_{i=1}^{n_j} \widehat{W}_{ij}T_{ij}}$	$E[Y_{ij}\|T_{ij} = 0, S_{ij} = j]$ $\uparrow$ $\frac{\sum_{i=1}^{n_j} (1 - T_{ij})Y_{ij}}{\sum_{i=1}^{n_j} (1 - T_{ij})}$

Challenge – Two-Step Estimation

- Sampling variability in weight cannot be ignored
    - The sampling variability in weight, estimated through multilevel logistic regressions in step 1, needs to be reflected in the sampling variability of site-specific effect estimates.
- Solution: treat the two-step estimators as a one-step generalized method of moments (GMM) estimator, which solves the moment functions stacked from both steps.

Step 3 & 4

- Step 3: Population average effect estimation
  - Take average over site-specific estimate estimates
- Step 4: Between-site variance estimation
  - Purge average within-site variance from between-site variance of site-specific effect estimates.

Step 5: Sensitivity analysis

- Omitted pretreatment covariates may cause bias, and there is a lack of adjustment strategies for post-treatment covariates.
- Sensitivity analysis may be conducted to assess the impact of omitted pretreatment and posttreatment confounders.
- Conceptualizing and investigating the relationships among multiple mediators may help with handling observed post-treatment covariates.

Analysis Results

	Average Effect		Between-Site Variance	
	Estimate	Effect Size	Estimate	95% Plausible Values
ITT on mediator (odds ratio)	2.39 (p<0.001)			
ITT on outcome	21.84 (SE=5.19, p<0.001)	0.12	635.44 (p=0.03)	(-27.57, 71.25)
Direct Effect	13.47 (SE=5.18, p=0.009)	0.07	597.97 (p=0.045)	(-34.46, 61.40)
Indirect Effect	8.38 (SE=1.40, p<0.001)	0.04	30.26 (p=0.05)	(-2.40, 19.16)
Correlation			0.03	

Strengths

Unlike multilevel SEM, our method

- can directly estimate not only the population average direct effect and indirect effect, but also the between-site variance of the effects. This enables researchers to investigate possible heterogeneity of mediation mechanisms across sites. Such empirical information is essential for improving future intervention designs.
- relaxes the assumption of no treatment-by-mediator interaction.
- greatly simplifies the outcome model specification and therefore avoids possible outcome model misspecifications.

Potential Concerns

- The proposed estimation procedure may not be optimal when sites sizes are extremely small.
- SUTVA needs to be relaxed if an individual's potential mediators and potential outcomes are each a function not only of one's own treatment assignment but also of other individuals' treatment assignments, or if an individual's potential outcomes could additionally be affected by other individuals' mediator values.